

## Learning outcomes and competences

At the end of the course unit the learner is expected to be able to

1. Perform the elementary operations of matrix algebra and solve matricial equations and identify special matrices
2. Check if a matrix is invertible and calculate its inverse.
3. Classify and solve systems of linear equations, in matrix notation, using the methods of Gauss, Gauss-Jordan and Cramer.
4. Calculate the determinant, the eigenvalues and eigenvectors of a square matrix
5. Identify if a set, V , can be given a structure of vector space over a field of the real numbers, R.

6 . Determine if a given subset $M$ of $V$ is a vector subspace of $V$ (over $R$ ).
7. Identify if a given transformation between two vector spaces is linear, and determine the matrix of a linear transformation using the canonical basis of both spaces.
8. Calculate the kernel and the range of a linear transformation.

## Prerequisites

Before the course unit the learner is expected to be able to
Perform the basic operations of elementary calculus.

## Course contents

Matrices. Determinants. Systems of Linear Equations. Vector Spaces. Linear Transformations. Eigenvalues and Eigenvectors.

## Course contents (extended version)

1. Matrices

- Definitions, terminology and notations.
- Matrix operations
- Rules of matrix arithmetic.
- Inverse of square matrix.

Triangular, diagonal and symmetric matrices.
2 Determinants

- Definition and properties of determinants.
- Evaluation of determinants by row reduction.
- Evaluation of determinants by cofactor expansion.
- Adjoint matrix.
- Calculating the inverse of an invertible matrix using the adjoint matrix.

3. Systems of Linear Equations

- Consistency of linear systems.
- Inverse, Gaussian elimination and Gauss-Jordan methods.
- Cramer's rule.
- Calculating the inverse of a matrix using Gauss-Jordan method.

4. Vector Spaces

- Definitions and examples.
- Vector subspace.
- Linear independence/dependence.

Basis and dimension.

- n-dimensional Euclidean vector space.
- Norm, dot product and projections.

Cross product in R^3.
Schwarz inequality.
5. Linear Transformations

- Definitions and examples.

Kernel and range of a linear transformation.
Matrix of a linear transformation.

- Invertibility.

6. Eigenvalues and Eigenvectors

- Definitions and examples.
- Characteristic polynomial
- Eigenspace.
- Matrix diagonalization.


## Recommended reading

1. Howard, A \& Rorres, C. (2014). Elementary Linear Algebra - Applications version (11th ed. ). Wiley.
2. Penney, R. C. (2008). Linear Algebra - Ideas and applications (3rd ed. ). Wiley.
3. Strang, G. (2005). Linear Algebra and its applications (4th ed.). Brooks Cole.
4. Trigo, J. A. (2004). Noções sobre matrizes e sistemas de equações lineares. Porto: FEUP edições.

## Teaching and learning methods

The themes will be presented and discussed throughout the classes, using the resolution of tasks to deepen them. There will be individual and group sessions outside class schedule to accompany the student's work. The use of software will be encouraged.

## Assessment methods

1. Alternative 1 - (Regular, Student Worker) (Final, Supplementary) - Practical Work - 20\%

Intermediate Written Test - 40\%

- Final Written Exam - 40\%

2. Alternative 2 - (Regular, Student Worker) (Final, Supplementary)

- Intermediate Written Test - 50\%
- Final Written Exam - 50\%

3. Alternative 3 - (Regular, Student Worker) (Supplementary, Special)

Final Written Exam - 100\%

## Language of instruction

Portuguese, with additional English support for foreign students.

## Electronic validation

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| $10-10-2022$ | $11-10-2022$ | $11-10-2022$ | $24-10-2022$ |

